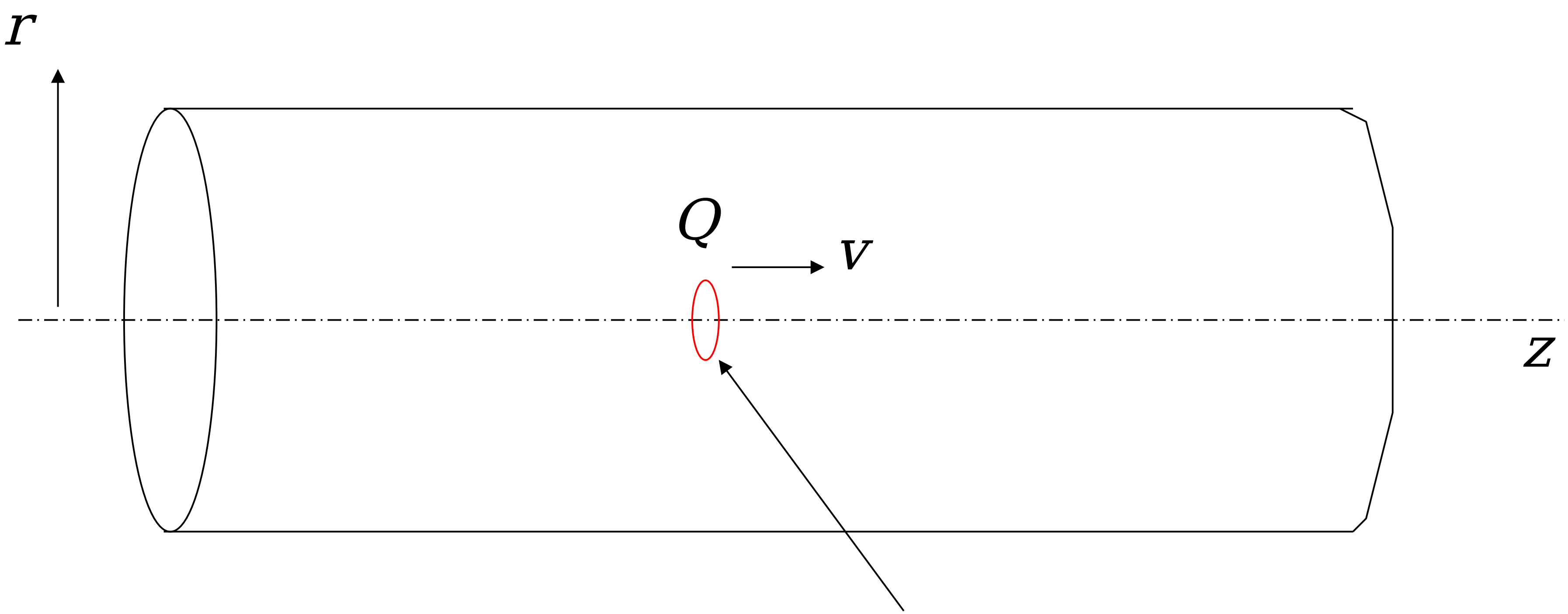
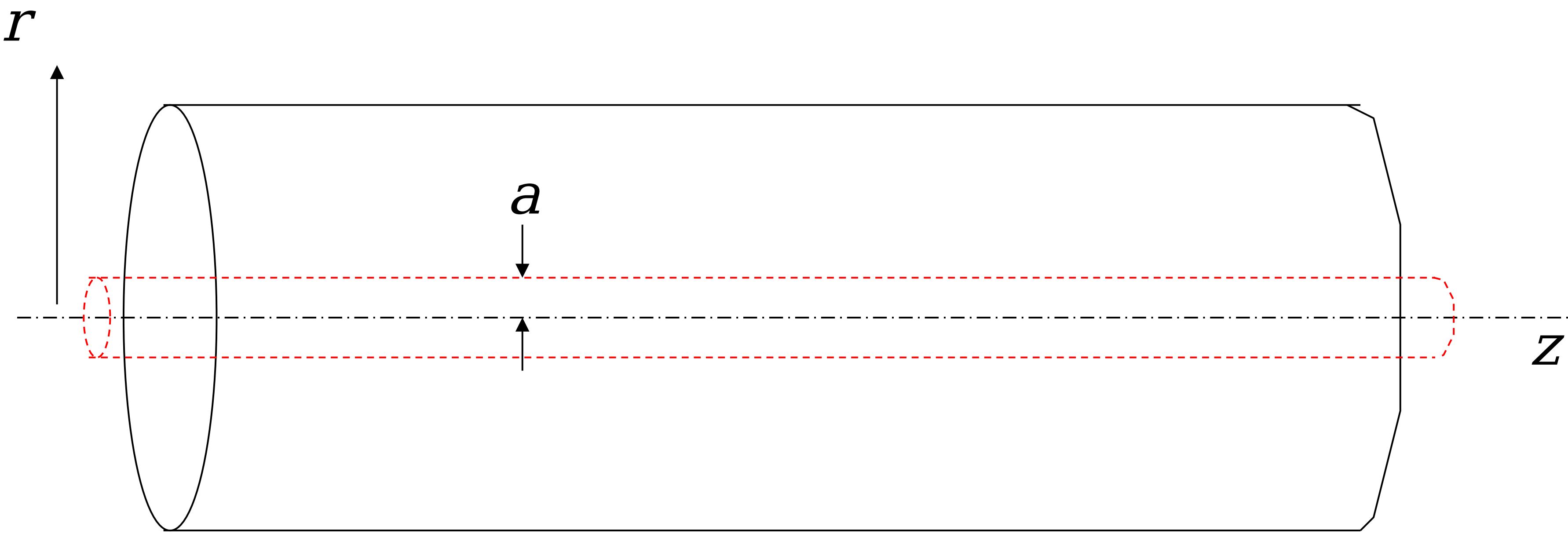
To compute the wake function, we consider ...

Ring of charge that generates EM field around it [2]



Dipole case:
- charge modulated by $\cos\ \theta$
- dipole moment $P = Qa$

Fourier transform with respect to t [3]



Charge density $$\rho(r,\theta,z) = \frac{P}{\pi a^2}\,\delta(r - a)\cos\theta\, e^{ikz}$$

NB: $v = \dfrac{\omega}{k}$ $\qquad c \neq \dfrac{\omega}{k}$ unless $v = c$

The case of v = c in vacuum

Region outside the beam pipe

Solution

$$E_z = \frac{A}{r}$$

$$cB_z = \frac{A}{r}$$

$$E_r = -ikA\ln r - \frac{B}{2r^2} + C$$

$$cB_r = -ikA\ln r + \frac{B}{2r^2} + C + \frac{iA}{kr^2}$$

$$E_\theta = ikA\ln r - \frac{B}{2r^2} - C$$

$$cB_\theta = -ikA\ln r - \frac{B}{2r^2} + C - \frac{iA}{kr^2}$$

A, B, C unknown constants

Physics of solution

When r → ∞, expect solution → 0

∴ Should drop ln r, so $A = 0$
and drop constant term C

$$E_z = 0$$

$$cB_z = 0$$

$$E_r = -\frac{B}{2r^2}$$

$$cB_r = \frac{B}{2r^2}$$

$$E_\theta = -\frac{B}{2r^2}$$

$$cB_\theta = -\frac{B}{2r^2}$$

Questions

- should E_z be zero?
- only one unknown, B
  - expect 2 for v < c (see [1])

o solve Maxwell's in cylindrical coordinates [2][4]

Each component of E or B $\quad R(r)\Theta(\theta)Z(z)T(t)$

Define

$R(r) \quad E_z \quad E_r \quad E_\theta \quad B_z \quad B_r \quad B_\theta$

Get

$\Theta(\theta) \quad \cos\theta \quad \cos\theta \quad \sin\theta \quad \sin\theta \quad \sin\theta \quad \cos\theta$

respectively, by inspection of Maxwell's.

$Z(z) \quad e^{ikz}$

$T(t) \quad e^{-i\omega t}$

Substituting into Maxwell's, get

$$\frac{\partial E_z}{\partial r} = -\frac{v}{r} B_z + ik\left(1 - v^2 \varepsilon \mu\right) E_r$$

Vanish in vacuum for v = c

$$\frac{\partial B_z}{\partial r} = -\frac{v \varepsilon \mu}{r} E_z + ik\left(1 - v^2 \varepsilon \mu\right) B_r$$

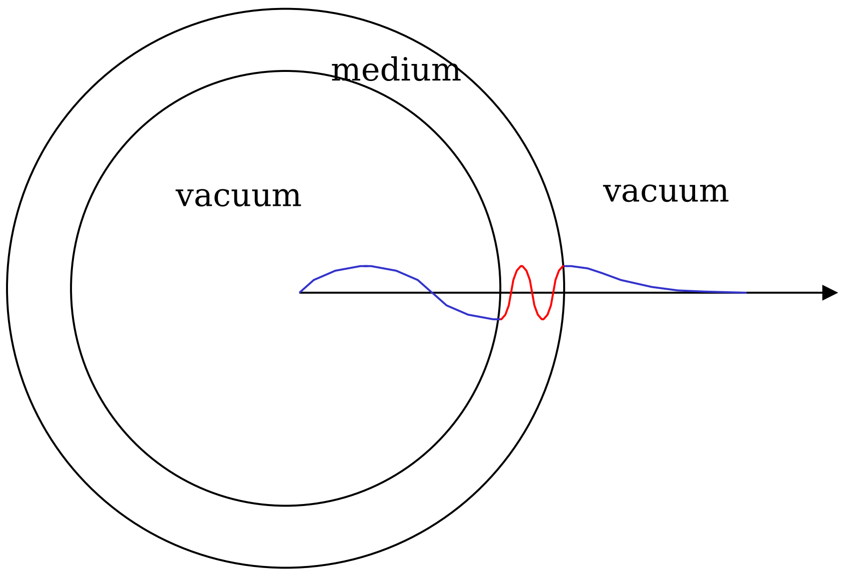
$$E_\theta = -vB_r + \frac{i}{kr} E_z$$

$$B_\theta = v\varepsilon\mu E_r - \frac{i}{kr} B_z$$

$$\frac{1}{r}\frac{\partial\left(rE_r\right)}{\partial r} - \frac{v}{r} B_r = \frac{P}{\pi \varepsilon v a^2}\delta(r-a) - i\left(k + \frac{1}{kr^2}\right) E_z$$

$$\frac{1}{r}\frac{\partial\left(rB_r\right)}{\partial r} - \frac{v\varepsilon\mu}{r} E_r = -i\left(k + \frac{1}{kr^2}\right) B_z$$

d to construct solutions and match them at boundaries [1][



Solutions for E_z

	v < c	v = c
vacuum	Modified Bessel	r, $1/r$
medium	Modified Bessel	Modified Bessel

References

[1] A. M. Al-Khateeb, et al, Transverse resistive wall impedances and shielding effectiveness for beam pipes of arbitrary
wall thickness, Phys. Rev. ST Accel. Beams 10, 064401 (2007)

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[2] Alex Chao, Physics of Collective Beam Instabilities in High Energy Accelerators (1993), pp. 4-6, 40-41, 51-52.

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[3] R. Gluckstern, CERN Yellow Report 2000-011 (2000), pp. 1-8.

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[4] B. Zotter, New Results on the Impedance of Resistive Metal Walls of Finite Thickness, CERN-AB-2005-043, pp. 1-4, 15-20.

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